Storage and retrieval of two unitary channels

M. Sedlák[1,2], A. Bisio[3], R. Stárek[4], N. Horová[4], M. Mičuda[4], J. Fiurášek[4]

[1] Research Center for Quantum Information, Institute of Physics, Slovak Academy of Sciences

[2] Faculty of Informatics, Masaryk University, Botanická 68a, 60200 Brno, Czech Republic

[3] Dipartimento di Fisica, Università di Pavia, via Bassi 6, 27100 Pavia, Italy

[4] Department of Optics, Faculty of Science, Palacký University, Olomouc, Czech Republic

Abstract

Ability to precisely capture the functioning of a state transformation is vital not only for quantum engineering. Single use of an entangled probe system can make a footprint with arbitrary precision, but the retrieval of the action of the stored transformation will either suffer from errors or will happen only with limited success probability. Thus, limits of storage and retrieval for n uses of one out of two unitary transformations are a quite fundamental task. We completely solve the problem for both maximum fidelity and perfect probabilistic approach for qubits. We present results of an optical experiment for the latter approach. The storage and retrieval quality have been assessed using quantum tomography of states and processes and the results are discussed in relation to non-optimal measure-and-prepare strategy to illustrate advantages of our protocol. The retrieval part of the experiment can be also interpreted as a realization of probabilistic quantum processor operating at an optimal trade-off between overlap of program states, distinctness of the desired unitary transformations and the achievable success probability. Finally, the optimal perfect probabilistic results are generalized to d dimensional case.

storage of quantum operations

General goal:

Apply an (unknown) quantum gate on an unknown target quantum state.

Catch:

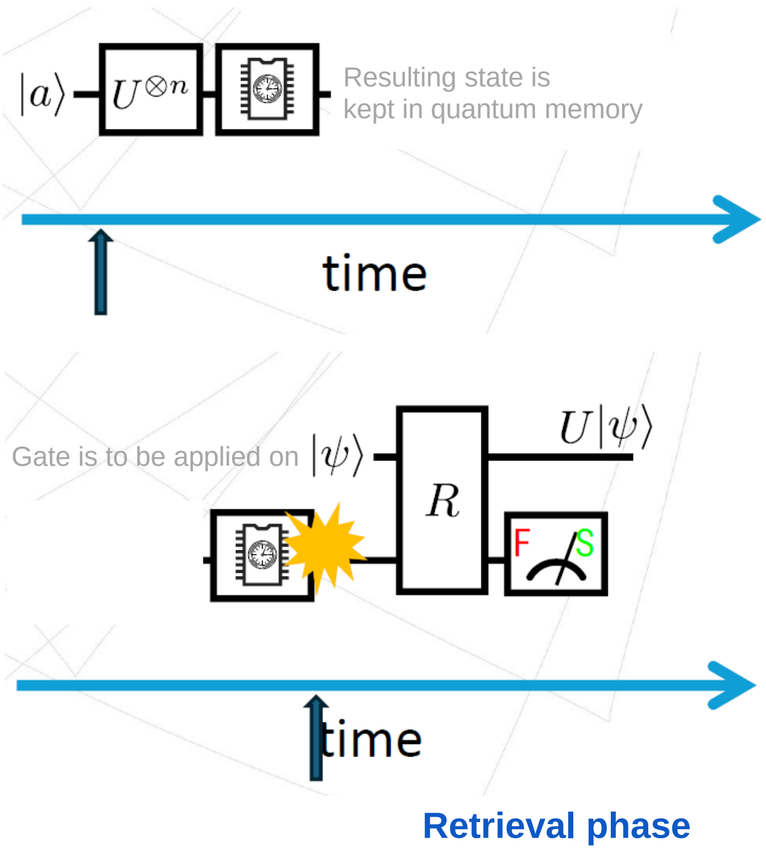
We lose access to the gate before the target state is prepared.

Solution:

Encode gate into state and store it in quantum memory and retrieve it at later time using a quantum programmable circuit.

gate storage & retrieval

Storage phase



Retrieval phase

Types of retrieval

Two types of retrieval were considered so far: **approximate deterministic** (this task was termed quantum learning [1]) or **perfect probabilistic** (denoted as storage and retrieval of unitary channels [2]). As the names suggest we either maximize average process fidelity, while requiring the retrieval to work every time, or we allow failure of the retrieval, but in the case of success we require perfect implementation of the unknown unitary U, respectively

N→1, 1-out-of-2 unitary storage

Storage:

• Stored unitary is without loss of generality either

$$U_0 = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix} \quad \text{or} \quad U_1 = \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{+i\alpha} \end{pmatrix}$$

both known and randomly selected in each run with equal probability. The choice is **unknown.**

• The stored operation can be applied N times during the storage phase to improve the performance

optimal storage

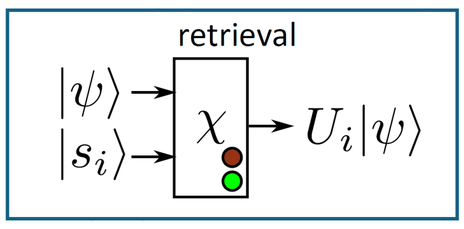
i = ?

$$|a\rangle \rightarrow \boxed{U_i^n} \rightarrow |s_i\rangle$$

Retrieval:

• quantum circuit programmed by state $|s_i\rangle$ can be understood as a programmable quantum processor.

• In perfect probabilistic version success heralds that U_i is perfectly applied onto target state $|\psi\rangle$.



$$\mathcal{R}_i(|\psi\rangle\langle\psi|) = \chi(|\psi\rangle\langle\psi| \otimes |s_i\rangle\langle s_i|)$$
$$\mathcal{U}_i(|\psi\rangle\langle\psi|) = U_i|\psi\rangle\langle\psi|U_i^\dagger$$

Process fidelity of deterministic retrieval

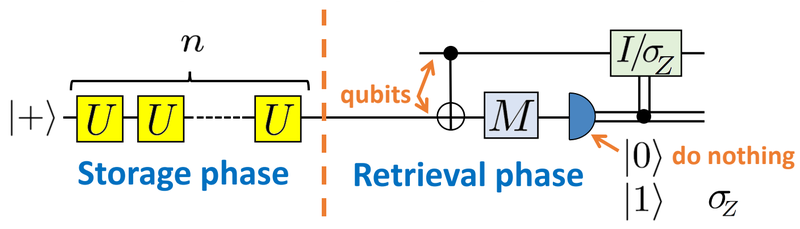
$$F = \tfrac{1}{2}F(\mathcal{R}_0,\mathcal{U}_0) + \tfrac{1}{2}F(\mathcal{R}_1,\mathcal{U}_1)$$

Optimization results in maximal value

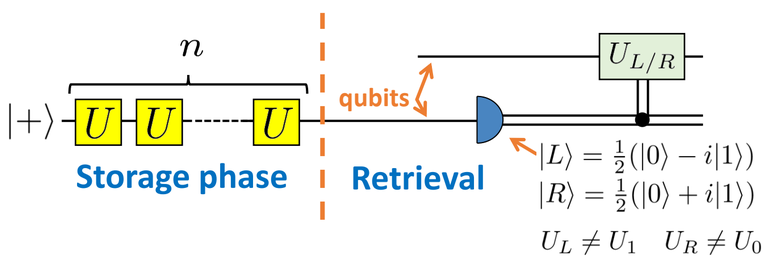
$$F = \tfrac{1}{2} + \tfrac{1}{2}\sqrt{1-(\sin 2\alpha \cos 2n\alpha)^2}$$

Final mathematical problem can be converted to minimum error discrimination problem for two pure states appearing with unequal prior probability, which is why the optimal value has the above form.

Coherent implementation of optimal protocol



Measure&prepare strategy is also optimal for det. retrieval



Maximal probability of perfect retrieval

Conditions for perfect retrieval

$$\mathcal{R}_0 = \lambda_0\,\mathcal{U}_0 \quad \mathcal{R}_1 = \lambda_1\,\mathcal{U}_1$$

λ_i is the conditional probability of success if unitary $\mathcal{U}_i$ was stored (i.e. stored state was $|s_i\rangle$)

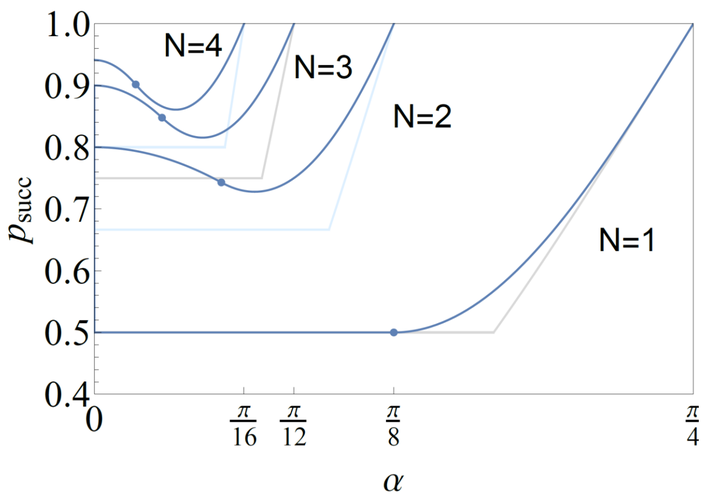
Figure of merit is the average success probability

$$F = \tfrac{1}{2}(\lambda_0 + \lambda_1)$$

Optimization can be rewritten as unambiguous discrimination problem for two pure states (UDP2P) appearing with unequal prior probability. For storage and retrieval only two out of three regimes of UDP2P are accessed, which is why the optimal formula splits into the following two expressions.

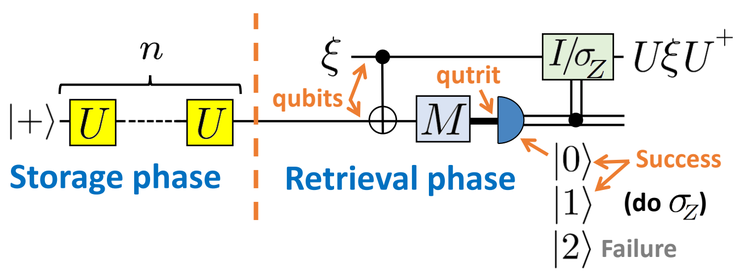
$$F = \begin{cases} 1 - \cos 2n\alpha \sin 2\alpha & \alpha \geq \alpha_{n,crit} \\ \frac{1-(\cos 2n\alpha)^2}{2(1-\cos 2n\alpha \cos 2\alpha)} & \alpha \leq \alpha_{n,crit} \end{cases},$$

Perfect retrieval results can be generalized also to the case of two d-dimensional unitaries. It turns out, that success probability depnds only on n and the angular spread of $U_1^\dagger U_0$.



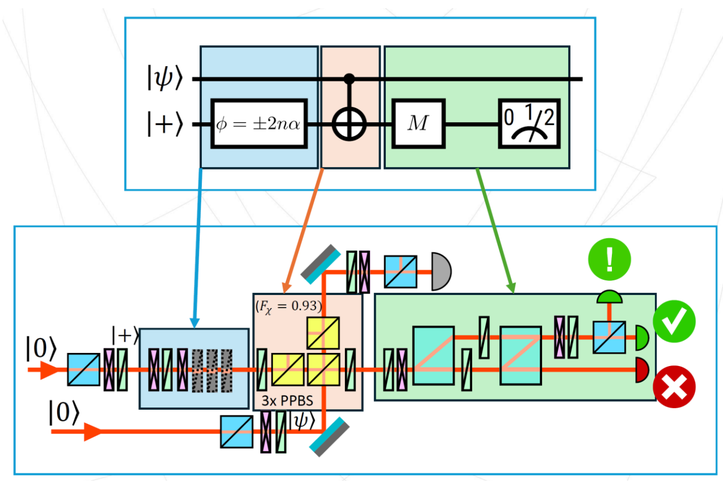
Optimal success probability of storage and retrieval of two d-dimensional unitary transformations as a function of n - the number of black box uses and of the angle α, which equals half of the angular spread of $U_1^\dagger U_0$. Fainted lines represent performance achievable by non-optimal strategies based on results from [3] and unambiguous discrimination. Small circles on the lines mark the transition between small/big alpha regime of the success probability.

Optimal Quantum circuit for perfect storage&retrieval of 2 unitaries

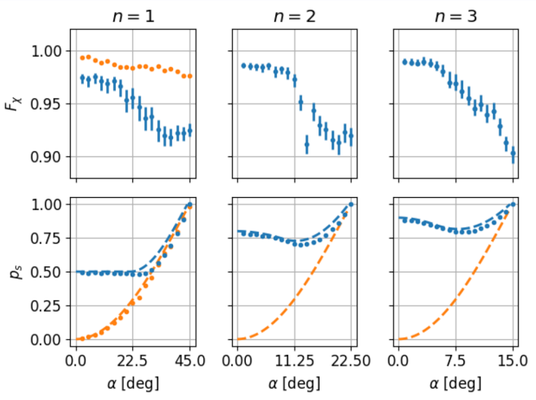


Quantum optics implementation of the optimal protocol

- Single-photon source CW SPDC, type-II
- Qubit-encoding: polarization
- Probabilistic CNOT heralded by detected coincidences
- Feed-forward is emulated by data mixing
- Retrieved operations are analyzed using channel tomography.



Results of the quantum optics experiment



Summary

This work derives optimal protocols for storage and retrieval of n-times used unknown unitary transformation, which was chosen from two known options. Unambiguous version of the problem is solved for arbitrary dimension and experimentally tested in quantum optics setup in the qubit case. Optimal protocol for maximum process fidelity approach does not need quantum memory, because it can be realized in a measure and prepare fashion.

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